Appendix F
Use of Logic Trees in Probabilistic Seismic Hazard Analysis

F-1. Logic Trees in Probabilistic Seismic Hazard

a. Introduction. The assessment of seismic hazards, whether on a deterministic or a probabilistic basis, must address uncertainties caused by an incomplete understanding of the mechanisms that control the complex process of earthquake generation and seismic wave propagation. Because of this uncertainty, seismic hazards must be assessed using assumptions about what constraints the available information provides on the location, size, and likelihood of occurrence of future earthquakes and their effects on a site. These uncertainties can be dealt with by a variety of approaches, ranging from simple engineering judgment to formal probabilistic treatment. Formal probabilistic treatment of uncertainty has the advantages of quantifying judgments that are always made in any assessment of seismic hazard and providing a framework for assessing the impact of new data or knowledge on an assessment.

b. Logic trees. Logic trees provide a convenient form for formal and quantitative treatment of uncertainty. The use of logic trees in probabilistic seismic hazard analysis has a long history, ranging from weighting of a few alternative assumptions (Cornell and Merz 1975; McGuire 1977; McGuire and Shedlock 1981) to full uncertainty treatment for all of the inputs to a probabilistic seismic hazard assessment (Kulkarni, Youngs, and Coppersmith 1984; Coppersmith and Youngs 1986; Electric Power Research Institute 1987; National Research Council 1988). Logic tree analysis consists of specifying a sequence of assessments that must be made in order to perform an analysis and then addressing the uncertainties in each of these assessments in a sequential manner. Thus, it provides a convenient approach for breaking a large, complex assessment into a sequence of smaller, simpler components that can be more easily addressed.

c. Structure. The general structure of a logic tree is shown in Figure F-1a. The logic tree is composed of a series of nodes and branches. Each node represents an assessment of a state of nature or an input parameter value that must be made to perform the analysis. Each branch leading from the node represents one possible discrete alternative for the state of nature or parameter value being addressed. If the variable in question is continuous, it can be discretized at a suitable increment. The branches at each node are intended to represent mutually exclusive and collectively exhaustive states of the input parameter. In practice, a sufficient number of branches are placed at a given node to adequately represent the uncertainty in the parameter estimation, as discussed in paragraph F-2.

F-2. Probabilities

a. Assigning probabilities. Probabilities that represent the relative likelihood or degree of belief that the branch represents the correct value or state of the input parameter are assigned to each branch. These probabilities are assessed conditionally on the assumption that all the branches leading to that node represent the true state of the preceding parameters. Because they are conditional probabilities for an assumed mutually exclusive and collectively exhaustive set of values, the sum of the conditional probabilities at each node is unity. The probabilities are usually based on subjective judgments because the available data are often too limited to allow for statistical analysis, and because scientific judgment is needed to weigh alternative interpretations of the available data. The logic tree approach simplifies these subjective assessments because the uncertainty in a single parameter is considered individually with all other parameters leading up to that parameter assessment assumed to be known with certainty. Thus, the nodes of the logic tree are sequenced to provide for the conditional aspects or dependencies among the

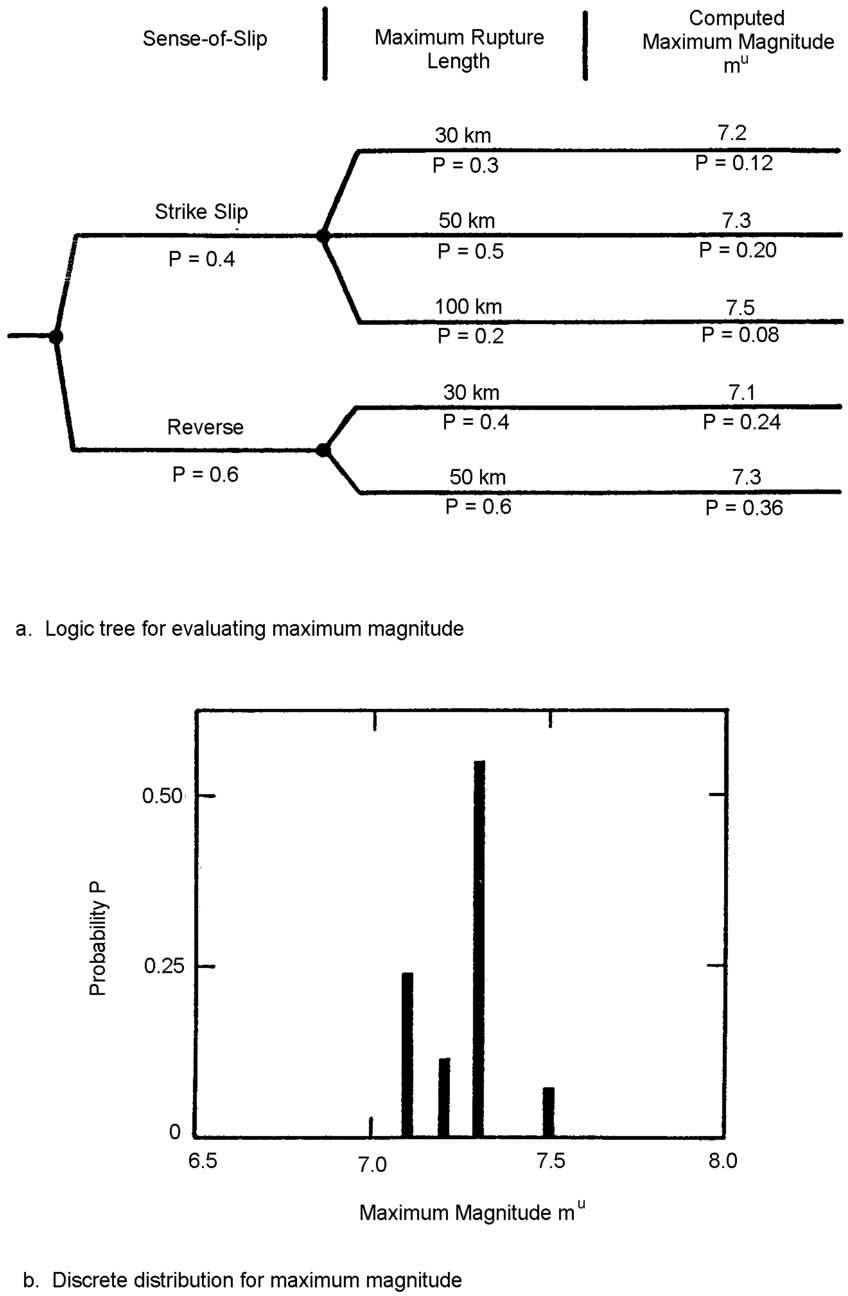

Figure F-1. Example logic tree for evaluating maximum magnitude

parameters and to provide a logical progression of assumptions from the general to the specific in defining the input parameters for an evaluation. In most cases, the probabilities assigned to the branches at a node are in units of tenths, unless there is a basis for more fine-scale resolution.

b. Types of probability assessments. Usually these weights represent one of two types of probability assessments.

(1) In the first, a range or distribution of parameter values is represented by the logic tree branches for that parameter and their associated weights. For example, the slip rate on a fault is usually uncertain because of uncertainties in the amount of displacement of a particular geologic unit across the fault and the age of the unit. The resulting slip rate is usually represented by a preferred value and a range of higher and lower values, similar to a normal or lognormal statistical distribution. This type of distribution can be represented by three (or more) branches of a logic tree. For example, Keefer and Bodily (1983) have shown that a normal distribution can be reliably represented by three values: the central estimate (with a weight of 0.6) and a higher and lower value (each with weights of 0.2) that represent the 5th and 95th percentiles (about plus or minus two standard deviations). Although a large number of branches for an individual assessment can be included on a logic tree, usually the results are not sensitive to having more than about three branches at any one node in a logic tree with many nodes.

(2) A second type of probability assessment to which logic trees are suited is in indicating a relative preference for or degree of belief in alternative hypotheses. For example, the sense of slip on a fault may be uncertain; two possible alternatives might be strike-slip or reverse-slip. Based on the pertinent data, a relative preference for these alternatives can be expressed by the logic tree weights. A strong preference is usually represented by weights such as 0.9 and 0.1 for the two alternatives. If there is no preference for either hypothesis, they are assigned equal weights (0.5 and 0.5 for two hypotheses). Increasing weights from 0.5 to 0.9 reflect an increasing preference for the alternative. Although the logic tree weights are ultimately subjective judgments based on available information, it is important to document the data and interpretations that led to the assessment of parameter values and to assignment of weights. The example logic tree shown in Figure F-1a might be used to represent the uncertainty in assessing the maximum magnitude for a fault on the basis of a relationship between maximum rupture length per event and earthquake magnitude (e.g., Slemmons 1982). In order to assess the maximum magnitude, two pieces of information are required: the sense of slip S of the fault and the maximum rupture length in any one event RL. The logic tree thus contains two levels of nodes, one for each parameter. In the example, the particular values that might be assigned to the maximum rupture length are dependent on the assumed sense of slip (strike-slip earthquakes may tend to produce greater rupture lengths than reverse earthquakes) and are thus more easily assessed given knowledge of the sense of slip. Consequently, the node for maximum rupture length per event is located after the node for sense of slip. The assigned weights reflect a slight preference for reverse faulting.

c. Maximum rupture length. The next level of assessment in the example addresses maximum rupture length for a maximum event. A range of possible values is considered for both assumptions about the sense of slip. The probability that 30 km is the correct maximum rupture length per event is assessed conditionally depending on which sense of slip is assumed to be correct. That is, the probability of a 30-km rupture length given strike-slip faulting, $P(RL=30 \mid S=\text{strike slip})$, is a separate assessment from $P(RL=30 \mid S=\text{reverse})$, and the two probabilities do not have to be equal. Similar assessments are made for the other branches at each node. In the example there is a preference for longer rupture lengths for strike-slip faulting than for reverse faulting represented by both the parameter values considered and the assessment of relative likelihoods. The logic tree shown in Figure F-1a defines a discrete distribution for the maximum magnitude computed using the relationship developed by Slemmons (1982). The resulting distribution is shown in Figure F-1b. The probability that the maximum magnitude, $m^u(S,RL)$, will take on any particular value $m^u(s_i, rl_j)$ is equal to the joint probability of the set of parameters s_i and rl_j being the true parameter values.

$$P[m^u(s_i, rl_j)] = P(S = s_i) \cdot P(RL = rl_j \mid s_i) \tag{F-1}$$

The expected or mean value of $m^u(S,RL)$ given the uncertainty in the input parameters S and RL is given by:

$$E[m^u\ (S,RL)] = \sum_i \sum_j m^u\ (s_i,rl_j) \cdot P(S = s_i) \cdot P(RL = rl_j|s_i) \tag{F-2}$$

and the variance in $m^u(S,RL)$ is given by:

$$VAR[m^u\ (S,RL)] = \sum_i \sum_j \left\{\left[(m^u(s_i,rl_j)\right] - E[m^u\ (S,RL)]\right\}^2 \cdot P(S = s_i) \cdot P(RL = rl_j|s_i) \tag{F-3}$$

In analyses with larger logic trees the results can be ordered to allow computation of various percentiles of the discrete distribution formed by the logic tree. (An example of a discrete distribution formed by a small logic tree is given in Example 1 of Appendix G.) Figure F-2 displays a partial logic tree representing a seismic hazard model developed for analysis of the seismic hazard at a site in the North Sea. The logic tree is laid out to provide a logical progression from general aspects/hypotheses regarding the characteristics of seismicity and seismic wave propagation in the region to specific input parameters for individual sources. The rationale for developing the various levels of the logic tree is discussed in paragraph F-3. The bases for selecting the parameter values and assigning relative weights are presented in Coppersmith and Youngs (1986).

F-3. Nodes

The first node of the logic tree represents the uncertainty in selecting the appropriate strong ground motion attenuation relationship. Attenuation was placed first in the tree because it is felt that a single relationship (whichever relationship may be “correct”) is applicable to all earthquake sources in the region. The second node of the logic tree represents the uncertainty in identifying what structures and processes are giving rise to earthquakes in the region. The fault model assumes the activity is occurring on reactivated normal faults that have been mapped using high-resolution seismic refraction and reflection surveys. The source zone model assumes that the sources of earthquakes are unknown except for their general extent as imaged by the historical seismicity. The next node applies to the fault source model only and addresses the question of differences in the rate of activity of the identified faults defined on the basis of differences in the age and amount of recent slip. The following nodes address the uncertainty in specifying the depth distribution of earthquake activity, the details of seismic zonation in the North Sea region, alternative constraints on earthquake recurrence parameters (b-value), and the appropriate relationship between earthquake magnitude and rupture size. All the levels of the logic tree to this point are assumed to apply universally to all sources. The logic tree is now expanded into subtrees to address parameters that vary independently from source to source. These include the sense of slip on individual sources, the dip of fault planes, and individual source maximum magnitudes. Each end branch of the logic tree shown in Figure F-2 defines a particular characterization of the seismic sources and ground motion attenuation in the region for which the rate of exceedance of ground motions at the site can be computed. The likelihood that this computation is the “correct” hazard at the site is given by the product of all the conditional probabilities along the path through the logic tree. The end branches thus define a discrete distribution for $\nu(z)$ (see Section III, Chapter 3).

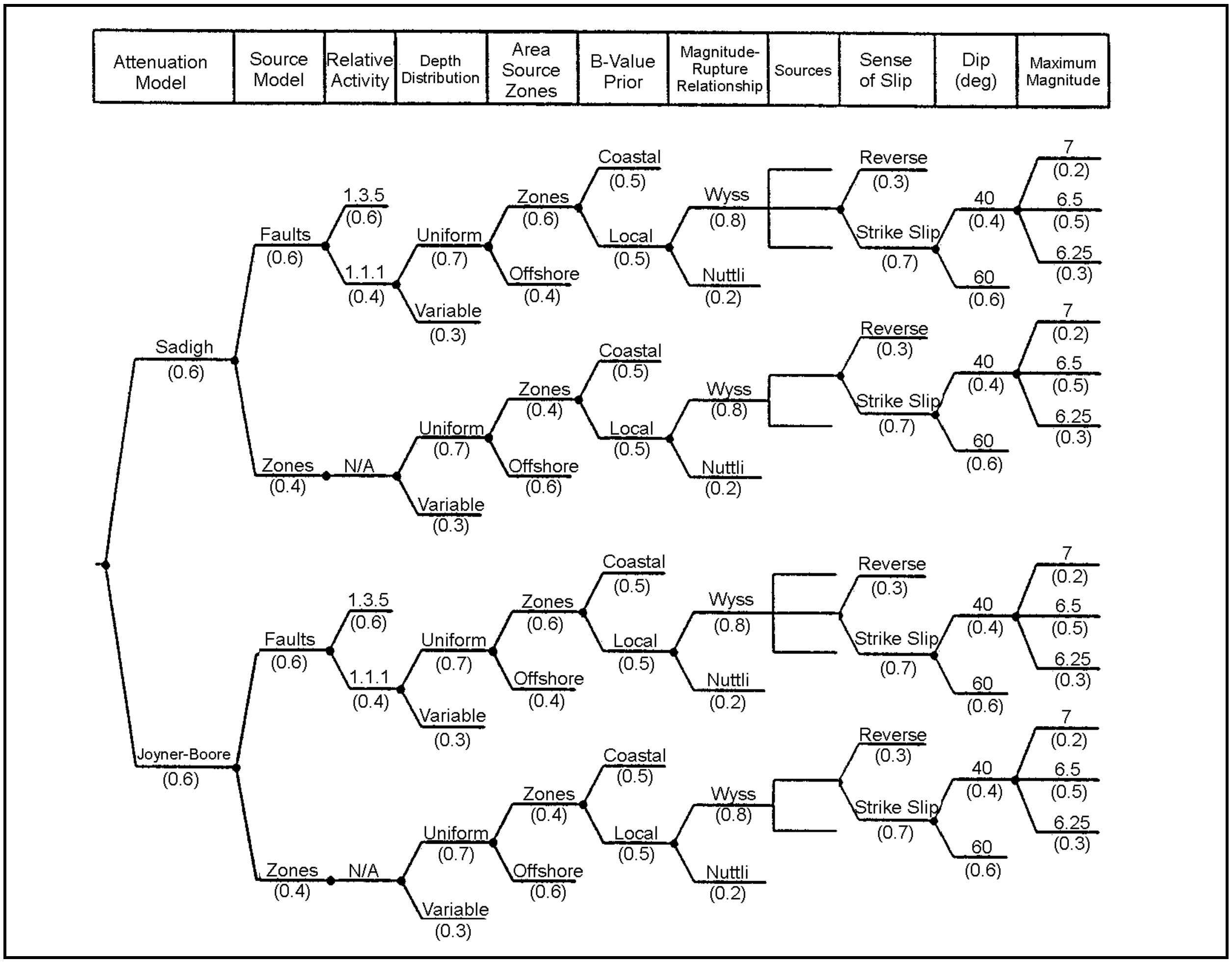

Figure F-2. Seismic hazard model logic tree for North Sea site